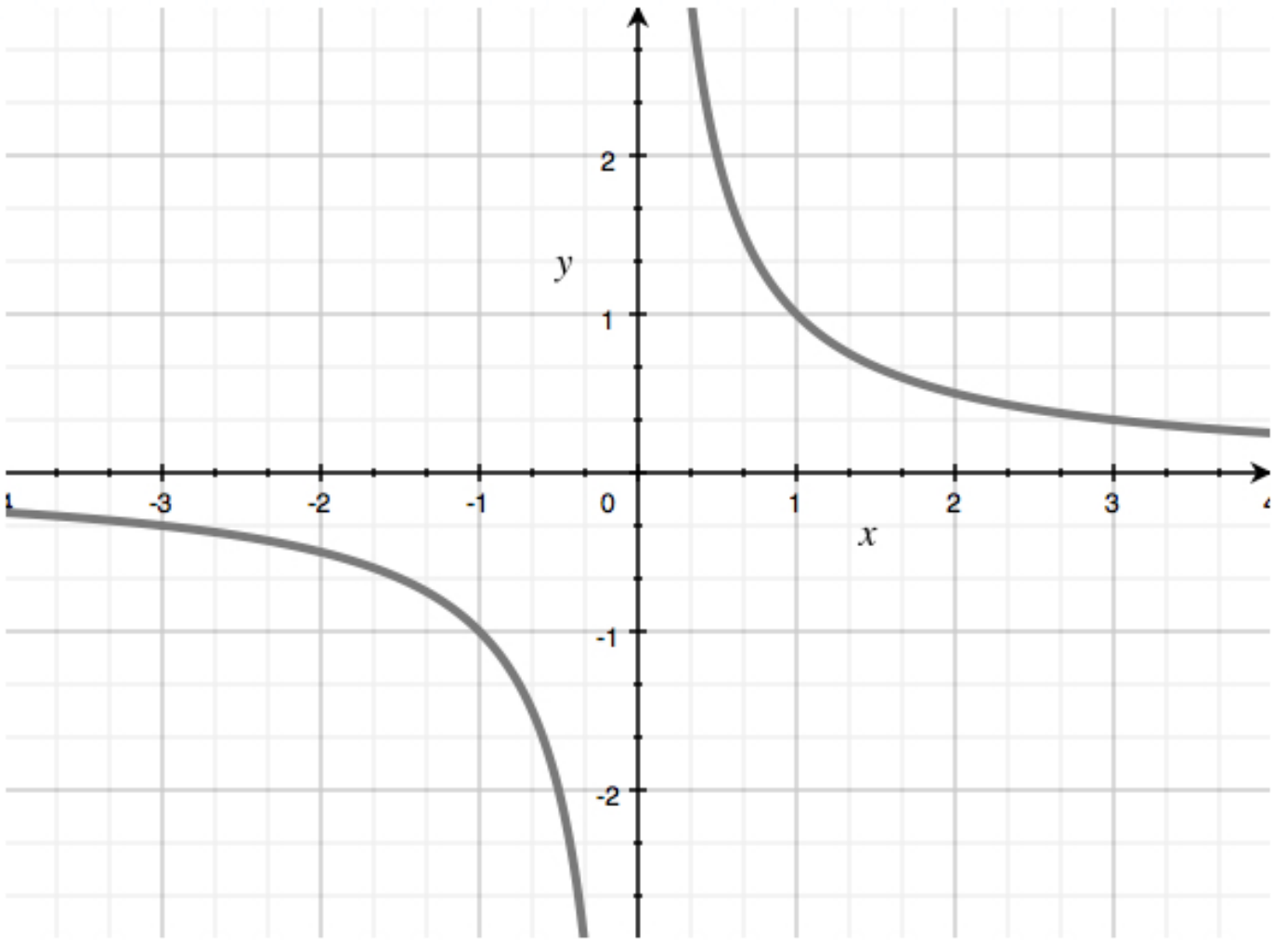
Infinite limits (vertical asymptotes)

Limits at infinity vs. infinite limits

Did you realize that there's a difference between *limits at infinity* and *infinite limits*? When you see "limits at infinity", it means the limit of the function as you approach positive or negative infinity. Contrast that with "infinite limits" which means that the value of a particular function is positive or negative infinity at a particular point.

In the graph below, the function has infinite, one-sided limits at $x = 0$. There's a vertical asymptote there, and we can see that the function approaches $-\infty$ from the left, and ∞ from the right. Talking about limits at infinity for this function, we can see that the function approaches 0 as we approach either ∞ or $-\infty$.



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x	$f(x)$
0	$\pm\infty$
$\pm\infty$	0

The first row of the chart above shows where the function has infinite limits. The second row shows the limits of the function at infinity.

How to find infinite limits

Infinite limits exist when we can plug in a number for x that causes the denominator of a rational function in lowest terms to equal 0.

Here is an example of a rational function in lowest terms, which means that we cannot factor and cancel anything in the fraction.

$$\lim_{x\to 1} \frac{1}{(x-1)^2}$$

We can see that setting $x = 1$ gives 0 in the denominator, which means that we have a vertical asymptote at $x = 1$, and therefore an infinite limit at that point.

Now that we've established that this is a rational function in lowest terms and that a vertical asymptote exists, all that's left to determine is whether the limit at $x = 1$ approaches ∞ or $-\infty$.

In order to do that, simply plug in a number very close to 1. If our result is positive, the limit will be ∞; if the result is negative, the limit is $-\infty$.

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$$\frac{1}{(0.99-1)^2}$$

$$\frac{1}{(-0.01)^2}$$

$$\infty$$

We can see that the result will be very large and positive, so we know that the limit of this function at $x = 1$ is ∞.

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